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Technical Notes

Computational Analysis of Mach Number Effects on the Edgetone Phenomenon

Taku Nonomura*
University of Tokyo, Kanagawa 229-8510, Japan
Hiroko Muranaka†
Tokyo College of Music, Tokyo 171-8540, Japan
and
Kozo Fujii‡
Japan Aerospace Exploration Agency, Kanagawa 229-8510, Japan

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I. Introduction

WHEN a jet impinges on a sharp edge, the flow near the edge is excited and emits an edgetone. The edgetone phenomenon is observed in some instruments, such as pipe organs and flutes, and some researchers [1] have attempted to use its mechanism to enhance fluid mixing. The basic characteristics of the edgetone phenomenon have been investigated in previous studies [2–15].

Powell [4,6] constructed a feedback-loop equation predicting the peak frequency of the edgetone:

$$f = \frac{N+p}{T_{\text{loop}}} = \frac{N+p}{T_1+T_2} \tag{1}$$

where f is the peak frequency, N is the oscillation mode number, p is the phase lag, T_{loop} is the duration of the feedback loop, T_1 is the time required for the jet disturbance to propagate from the nozzle exit to the edge, and T_2 is the time required for the acoustic wave to propagate from the edge to the nozzle exit, which is defined as follows:

$$T_1 = \int_0^L \frac{\mathrm{d}x}{U_c} \tag{2}$$

$$T_2 = \frac{L}{c} \tag{3}$$

where U_c is the jet-disturbance convective velocity at the jet center, similar to previous studies [6,7], L is the distance from the nozzle exit to the edge, and c is the sound speed.

Based on theoretical considerations, Powell [4,6] made two assumptions for the practical use of Eq. (1):

1) The speed of the jet disturbance from the nozzle exit to the edge is constant.

2) The phase lag p is constant with a value of 0.25.

The frequency predicted by Powell's feedback-loop equation with the above assumptions agreed well with that from the experimental data [6]. However, this feedback loop itself has not been quantitatively verified by calculating the constant phase lag p in various cases based on the measured T_1, T_2, and f. It is still important to verify this edgetone mechanism for advancing instrument design and for developing its various applications.

The feedback loop of the edgetone can be verified by investigating the effect of Mach number. If Powell's feedback-loop equation in the following nondimensional form is correct, the Strouhal number Sr of the peak frequency of the same mode decreases with increasing jet Mach number:

$$Sr = f\frac{L}{U_J} = \frac{N+p}{T_{\text{loop}}}\frac{L}{U_J} = \frac{N+p}{T^*_{\text{loop}}} = \frac{N+p}{T^*_1+T^*_2} = \frac{N+p}{T^*_1+M_J} \tag{4}$$

where

$$T^*_{\text{loop}} = T_{\text{loop}}\frac{U_J}{L} \tag{5}$$

$$T^*_1 = T_1\frac{U_J}{L} = \int_0^L \frac{\mathrm{d}x}{U_c}\frac{U_J}{L} \tag{6}$$

$$T^*_2 = T_2\frac{U_J}{L} = \frac{L}{c}\frac{U_J}{L} = M_J \tag{7}$$

U_J is the averaged jet velocity at the nozzle exit, and $M_J = U_J/c$ is the jet Mach number at the nozzle exit. Here, T^*_1 seems to be approximately constant for a fixed Reynolds number.

During experiments, it is difficult to vary the jet Mach number while maintaining a fixed Reynolds number, since increasing the jet speed simultaneously increases the Reynolds number. In fact, Krothapalli et al. [1] experimentally studied the effects of Mach number on the peak frequency of the edgetone, but the result included a change in the Reynolds number. Additionally, they did not measure jet-disturbance convection which is required for the quantitative verification of Powell's feedback-loop equation.

In computational studies, it is easy to change the jet Mach number while easily keeping the Reynolds number fixed. Therefore, the computational approach is more appropriate to verify the feedback-loop mechanism. In this study, the edgetone phenomena of a two-dimensional laminar jet and a triangle wedge are computed at Mach numbers ranging from 0.087 to 0.435, at three fixed Reynolds numbers: 208, 416, and 624. These conditions are highly idealized for the verification of Powell's feedback-loop equation and correspond to experiments of micro-sized models with air. Moreover, the fixed Reynolds number and varying Mach number are realized by changing the model size. The effects of Mach number on the peak frequency of the edgetone are discussed in light of the results. Then the phase lag p is investigated for the quantitative verification of Powell's feedback-loop equation. More details can be found in [16].

II. Computational Condition

The working fluid is assumed to be air. The coordinate system and notation of the geometric parameters are shown in Fig. 1. In this

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*Graduate Student, Department of Aeronautics and Astronautics, Sagamihara; currently Postdoctoral Researcher, Institute of Space and Astronautics Science, Japan Aerospace Exploration Agency. Member AIAA.

†Associate Professor, Department of Education, 3-4-5 Minamiikebukuro, Toshima.

‡Professor, Department of Space Transportation Engineering, Institute of Space and Astronautical Science, 3-1-1, Yoshinodai, Sagamihara. Fellow AIAA.

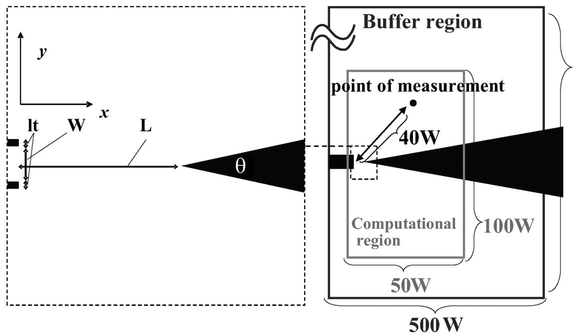

Fig. 1 Computational geometry.

study, L/W is set to 5, θ is set to 20 deg, and lt/W is set to 0.017. The jet profile is set to a top hat type, where the normalized momentum thickness $2\phi/W^{-1}$ is set to 0.05. The jet Mach numbers $M_{J\max}$ are 0.087, 0.174, 0.261, 0.348, or 0.435. The Reynolds number, based on the averaged jet velocity and nozzle width, is 208, 416, or 624. In total, 15 cases were computed.

III. Numerical Method

A. Computational Scheme

The two-dimensional compressible Navier–Stokes equations in the generalized coordinate system are solved by a well-validated computational code presented in [17–19]. The sixth-order compact difference scheme [20] is employed for spatial derivatives. A sixth-order tridiagonal filter with filter coefficient $\alpha_f = 0.42$ is used to suppress numerical instabilities [21]. Jameson–Baker's four stage second-order Runge–Kutta scheme [22] is adopted for time integration.

The total number of grid points is approximately 100,000. We conducted a grid convergence study with three different grid resolutions (60,000 to 100,000 grid points), and the strongest peak remains constant for the grids tested. The grid cutoff Strouhal number is 2.87 in the case of jet Mach number 0.435, and 15.2 in the case of jet Mach number 0.087. Buffer regions [23] are employed to avoid nonphysical reflection of the acoustic waves (Fig. 1).

The maximum Courant–Friedrichs–Lewy number is approximately 1 and more than 600,000 steps are integrated. No disturbances are introduced in the computation except for the initial 1000 steps of impulsive start. For the initial 1000 steps, we added a y-direction velocity disturbance (0.1% of jet velocity, constant along y) at the nozzle exit.

B. Postprocessing

Sound waves are directly computed without acoustic analogies. The sound pressure is measured at a point that is $40W$ from the edge, as shown in Fig. 1. Near the measurement point, the pressure fluctuation is proportional to $r^{-1/2}$ as expected for a two-dimensional wave.

The jet disturbance is defined as a y-velocity disturbance $v(x,t)$ at the center of the jet. Using $v(x,t)$, the convective velocity U_c is computed as follows, where $v(x,t)$ is assumed to be defined as

$$v(x,t) = A(x)\sin(\omega t + \phi(x)) \tag{8}$$

The disturbance moves along the lines where the argument of the sine function in Eq. (8) is constant:

$$\omega t + \phi(x) = \text{const.} \tag{9}$$

From this equation, $U_c(x) = \mathrm{d}x/\mathrm{d}t$ is computed as

$$U_c(x) = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}\phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} = \omega\frac{\partial x}{\partial \phi} = \omega\frac{1}{\frac{\partial \phi}{\partial x}} \tag{10}$$

This assumption is the same as that of Karamcheti et al. [7]. In this study, U_c is numerically calculated using Eq. (10).

IV. Results

The Strouhal numbers of the strongest peak frequencies of the edgetone are plotted against the Mach number in Fig. 2. Cases in which no edgetone is generated are not plotted. The characteristic flowfields of modes 1 and 2 and flowfields without edgetone excitation are also shown. The obtained flowfields correspond to those observed experimentally [2]. For a Reynolds number of 208, a peak of mode 1 (around $Sr = 0.5$) at Mach number 0.087 is observed. At this Reynolds number, no edgetone is generated when the Mach number is high. For a Reynolds number of 416, peaks of mode 1 (around $Sr = 0.5$) at Mach numbers 0.087, 0.174, and 0.261 are observed. The Strouhal number of the peak frequency decreases with increasing Mach number, as expected, and no edgetone is generated in the case of Mach numbers 0.348 and 0.435. For a Reynolds number of 624, peak of mode 1 (around $Sr = 0.5$) at Mach number 0.087 and peaks of mode 2 (around $Sr = 1.0$) at Mach numbers 0.174, 0.261, 0.348, and 0.435 are observed. The Strouhal number of the peak frequency at the same mode decreases with increasing Mach number, similar to the case of a Reynolds number of 416.

The following can be observed as the jet Mach number increases: 1) Strouhal number of the peak frequency at the same mode decreases, as expected; 2) an edgetone is not generated; and 3) higher oscillation modes are excited. The first observation seems to be related to the Powell's feedback-loop equation and corresponds to experimental observation [1] for high Reynolds number. The second

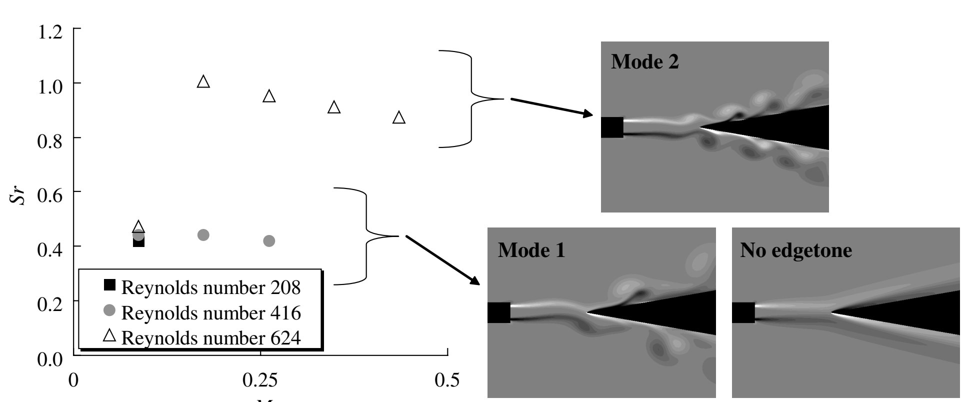

Fig. 2 Strouhal number of the strongest peak frequencies of edgetone against Mach number, and characteristic flowfields for each mode. Instantaneous vorticity distribution for each flowfield is shown.

Table 1 Computational results applied to Powell's feedback-loop equation

Reynolds number	Mach number	N	Sr	T_1^*	T_2^*	$N+p$	p
208	0.087	1	0.421	1.890	0.087	0.83	−0.17
208	0.174	No edgetone	——	——	——	——	——
208	0.261	No edgetone	——	——	——	——	——
208	0.348	No edgetone	——	——	——	——	——
208	0.435	No edgetone	——	——	——	——	——
416	0.087	1	0.441	1.705	0.087	0.79	−0.21
416	0.174	1	0.441	1.674	0.174	0.81	−0.19
416	0.261	1	0.421	1.673	0.261	0.81	−0.19
416	0.348	No edgetone	——	——	——	——	——
416	0.435	No edgetone	——	——	——	——	——
624	0.087	1	0.474	1.639	0.087	0.82	−0.18
624	0.174	2	1.006	1.614	0.174	1.80	−0.20
624	0.261	2	0.952	1.642	0.261	1.81	−0.19
624	0.348	2	0.912	1.634	0.348	1.81	−0.19
624	0.435	2	0.874	1.644	0.435	1.81	−0.19

characteristic seems to be related to the decrease in shear-layer growth rate due to high Mach number which is studied by a linear stability analysis [24]. The third observation could be due to the decrease in peak frequency with increasing Mach number (observation 1), leading to a change of the mode with a peak frequency near the most unstable frequency of the shear layer.

V. Verification of Powell's Feedback-Loop Equation

The phase lag p in Powell's feedback-loop equation is investigated. If the phase lag p is constant, the feedback-loop equation seems to be valid quantitatively because the phenomenon can be explained by the feedback loop. The phase lag p is calculated from Eq. (4) by substituting the computed results of T_1^* and using Eq. (7) for T_2^*.

N, $N+p$, the Strouhal number of the peak frequency Sr, T_1^* and T_2^* are summarized in Table 1. Figure 3 shows the relation between Sr and $1/T^*_{\mathrm{LOOP}}$ obtained from the computational results, where the three lines show the results for $p=0$, 0.25 and -0.2 in Eq. (4). Table 1 and Fig. 3 clearly show that the phase lag computed with the present results is constant and approximately equal to -0.20. Considering the phase lag p in the Powell's equation does not seem to change theoretically for similar flowfields, the fact that the computed phase lag p is almost constant shows that Powell's feedback-loop equation is quantitatively valid. Also, the present computational results show that the phase lag p is smaller than in Powell's assumption. However, this phase lag p was calculated with only the data from the jet center. If we use the data inside the shear layer, the value changes. Therefore, the value of the phase lag p is not important for verifying Powell's equation.

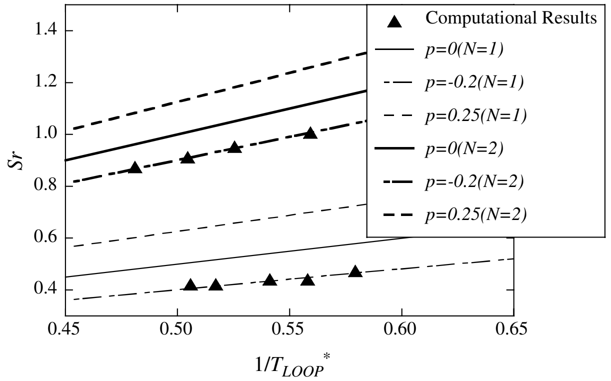

Fig. 3 Relation between the Strouhal number and duration of the feedback loop in the present results (triangle) and Powell's feedback equation (lines) assuming the phase lag p is 0, −0.2, and 0.25.

VI. Conclusions

The effects of Mach number on the edgetone were investigated, and the mechanism of the feedback loop of the edgetone phenomenon was verified using a high-order two-dimensional CFD code. The computational results show that when the jet Mach number increases, the Strouhal number of the peak frequency of the same mode decreases, as expected. This indicates that the edgetone mechanism is driven by the feedback loop. Powell's feedback-loop equation is verified using the present computational results. The computed results show that the phase lag p is almost constant for a wide range of Reynolds numbers and jet Mach number parameters. Therefore, Powell's feedback-loop equation is found to be quantitatively valid.

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T. Lyrintzis
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